Indian Statistical Institute
II Semestral Examination 2008-2009
B.Math.Hons.III year
Combinatorics and Graph Theory

Date:01-05-2009 Duration: 3 Hours Instructor: N.S.N.Sastry

Max Marks 100

Answer all questions. Your answer should be complete.

1. Define a strongly regular graph. Show that the complement of a strongly regular graph is also strongly regular.

   Let G be a finite rank 3 permutation group of even order. Construct a non-trivial strongly regular graph admitting G as an automorphism group. Justify your answer. $[2 + 5 + 5]$

2. a) Define the expanding constant of a k-regular, connected, finite graph.

   b) Show that it is at least $\frac{k-\mu_1}{2}$, where μ_1 is the largest nontrivial eigen value of the adjacency matrix of the graph.

   c) Show that, if h_n denotes the expanding constant of a cycle on n vertices, then $h_n \longrightarrow 0$ as $n \longrightarrow \infty$. $[2 + 9 + 6]$

3. a) Show that a 4-arc in a projective plane π of order 4 is contained in a hyperoval of π.

   b) Use this to compute the number of hyperovals in a projective plane of order 4. $[7 + 3]$

4. Show that a projective plane of order 4 is unique, up to isomorphism. $[10]$

5. Define a perfect linear code. Define a q-ary Hamming code. Compute its parameters. $[4 + 4 + 6]$

6. Define a Hadamard matrix of order n. Show that there is a Hadamard matrix of order $q + 1$ for each prime power q such that $q \equiv 3 \pmod 4$. $[2 + 12]$

7. a) State the Bruck-Ryser theorem on the existence of a projective plane of order n. Deduce that there exists no projective plane of order 6. $[2 + 6]$

8. a) Show that the multiplicative group of a finite field is cyclic.

   b) Show that a finite field of order p^n, p a prime, has a subfield of order p^s if, and only if, s divides n. $[7 + 8]$

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